Dimension بعد
unit واحد
$\frac{Length}{time}$ ← سرعت
$\frac{m}{s}$

Chapter 2. Units

آحاد

اغلب هرگاه می‌توانید اندازه‌گیری سنجیدن بیان اعداد

> *I often say that when you can measure what you are speaking about, and express it in numbers, you know something about it; but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meagre and unsatisfactory kind; it may be the beginning of knowledge, but you have scarcely, in your thoughts, advanced to the stage of science, whatever the matter may be*
> – William Thomson (Lord Kelvin)

ضعیف، ناکافی
ناراضی ← نوع شناخت به‌ندرت افکار پیشرفت
مرحله ← علم

اندازه وزن سرعت و غیره
اشیا عملکرد هدف انجام تبدیل بالعکس عنوان چالشی

All engineered systems require measurements for specifying the size, weight, speed, etc. of objects as well as characterizing their performance. Understanding the application of these units is the single most important objective of this textbook because it applies to all forms of engineering and everything that one does as an engineer. Understanding units is **far** more than simply being able to convert from feet to meters or vice versa; combining and converting units from different sources is a challenging topic. For example, if building insulation is specified in units of BTU inches per hour per square foot per degree Fahrenheit, how can that be converted to thermal conductivity in units of Watts per meter per degree C? Or can it be converted? Are the two units measuring the same thing or not? (For example, in a new engine laboratory facility that was being built for me, the natural gas flow was insufficient… so I told the contractor I needed a system capable of supplying a minimum of 50 cubic feet per minute (cfm) of natural gas at 5 pounds per square inch (psi). His response was "what's the conversion between cfm and psi?" Of course, the answer is that there is no conversion; cfm is a measure of flow rate and psi a measure of pressure. One might as well be asking what's the conversion between kilograms and miles.) Engineers must struggle with these misconceptions every day.

اندازه‌گیری امکانات آزمایشگاهی
حجم
دبی گاز
cfm : $\frac{ft^3}{min}$
فشار
psi : $\frac{lb}{in^2}$
دبی جریان اندازه‌گیری فشار
سروکله زدن
این
مبارزه کردن، نبرد، کشمکش
مفهوم اشتباه، سوء برداشت، تصور غلط

Base units

تحمیل کردن، زیر بار، تحت فشار

Engineers in the United States are burdened with two systems of units and measurements: (1) the *English* or *USCS* (US Customary System) ☹ and (2) the *metric* or *SI* (Système International d'Unités) ☺. Either system has a set of *base units* , that is, units which are defined based on a standard measure such as a certain number of wavelengths of a particular light source. These base units include:

سیستم مرسوم، متعارف، عرف
سیستم بین‌المللی
مجموعه
مثل

- Length (meters (m), centimeters (cm), millimeters (mm); feet (ft), inches (in), kilometers (km), miles (mi))
    - 1 m = 100 cm = 1000 mm = 3.281 ft = 39.37 in
    - 1 km = 1000 m
    - 1 mi = 5280 ft
- Mass (lbm, slugs, kilograms); (1 kg = 2.205 lbm = 0.06853 slug) (lbm = "pounds mass")
- Time (seconds; the standard abbreviation is "s" not "sec") (same units in USCS and SI!)
- Electric current (really electric charge in units of coulombs [abbreviation: 'coul'] is the base unit and the derived unit is current = charge/time) (1 coulomb = charge on 6.241506 x 10^{18} electrons) (1 ampere [abbreviation: amp]= 1 coul/s)

is equal to ، equals ، is
Point : ممیز
oh
آمپر : آمپر
کولوم → per

Moles are often reported as a fundamental unit, but it is not; it is just a bookkeeping convenience to avoid carrying around factors of 10^{23} everywhere. The choice of the number of

دفترداری
راحتی، آسودگی خودداری حمل کردن
Ten to the power of 23

دلخواه، اختیاری، قراردادی

ذره قراردادی تعریف

particles in a mole of particles is completely arbitrary; by convention Avogadro's number is defined by $N_A = 6.0221415 \times 10^{23}$, the units being particles/mole (or one could say individuals of any kind, not limited just to particles, e.g. atoms, molecules, electrons or students).

واحد مشتق شده

Temperature is frequently interpreted as a base unit but again it is not, it is a *derived unit*, that is, one created from combinations of base units. Temperature is essentially a unit of energy divided by Boltzman's constant. Specifically, the average kinetic energy of an ideal gas particle in a 3-dimensional box is 1.5kT, where k is Boltzman's constant = 1.380622×10^{-23} J/K (really (Joules/particle)/K; every textbook will state the units as just J/K but you'll see below how useful it is to include the "per particle" part as well). Thus, 1 Kelvin is the temperature at which the kinetic energy of an ideal gas (and **only** an ideal gas, not any other material) molecule is 1.5kT =2.0709×10^{-23} J.

متناوب ترجمه

per

انرژی جنبشی

The ideal gas constant ($\Re$) with which you are very familiar is simply Boltzman's constant multiplied by Avogadro's number, *i.e.*

$$\Re = kN_A = \left(1.38\times10^{-23}\frac{J}{particle\ K}\right)\left(6.02\times10^{23}\frac{particle}{mole}\right) = 8.314\frac{J}{mole\ K}\frac{cal}{4.184J} = 1.987\frac{cal}{mole\ K} \qquad \text{(Equation 1)}$$

In the above equation, note that we have multiplied and divided units such as Joules as if they were numbers; this is valid because we can think of 8.314 Joules as 8.314 x (1 Joule) and additionally we can write (1 Joule) / (1 Joule) = 1. Extending that further, we can think of (1 Joule)/(1 kg m^2/s^2) = 1, which will be the basis of our approach to units conversion – multiplying and dividing by 1 written in different (and sometimes odd-looking) forms. Note also the value of the "hidden unit" 'particle' in the above equation. I find it extremely useful to include such units because the real units aren't J/K; if you have 2 particles you'll have twice as much energy (J) at the same value of K (temperature), so the real units ARE in fact J/(particle K).

Why does this discussion apply only for an ideal gas? By definition, ideal gas particles have only kinetic energy and negligible potential energy due to inter-molecular attraction; if there is potential energy, then we need to consider the total internal energy of the material (E, units of Joules) which is the sum of the microscopic kinetic and potential energies, in which case the temperature for any material (ideal gas or not) is defined as

$$T = \left(\frac{\partial U}{\partial S}\right)_{V=const.} \qquad \text{(Equation 2)}$$

ارعاب کننده بحرانی

where U is the internal energy of the material (units J), S is the entropy of the material (units J/K) and V is the volume. This intimidating-looking definition of temperature, while critical to understanding thermodynamics, will not be needed in this course. (Until you read this you thought you understood temperature because of its common usage and a handy device called a thermometer; in fact, temperature is quite difficult to understand. The one thing you should understand is that it's the driving force for heat transfer, that is, heat must always flow from a higher to a lower temperature and never the reverse.)

نیروی محرکه برعکس

2*3: Multiply two by three

$\frac{2}{3}$: Two divided by three

10^{-23} { Ten to the Power of minus 23
10 to the minus 23

4+6=10 { four plus six equals ten
The sum of 6 and 4 is 10

6-2=4 { Six minus two is four
Subtract two from six is four
Take two away from six is four

6*4=24 { Six multiplied by four equals 24
Six times four equals 24
The product of six and four is 24
Multiply six by four is 24

15÷3=5

Fifteen divide by three equals five

Divide Fifteen by three is five

finite محدود
infinite نامحدود

Derived units

واحدهای مشتق شده

ترکیب

Derived units are units created from combinations of base units; there are an infinite number of possible derived units. Some of the more important/common/useful ones are:

ممکن مهم معمول مفید

قبل از نقطه، ویرگول ها، دونقطه: فاصله نباشه
بعدش اگر جمله ادامه داشت فاصله نیاز است.

- Area = length^2; 640 acres = 1 mile^2, or 1 acre = 43,560 ft^2
- Volume = length^3; 1 ft^3 = 7.481 gallons = 28,317 cm^3; also 1 liter = 1000 cm^3 = 61.02 in^3
- Velocity = length/time
- Acceleration = velocity/time = length/time^2 (standard gravitational acceleration on earth = g = 32.174 ft/s^2 = 9.806 m/s^2)
- Force = mass * acceleration = mass*length/time^2
    - 1 kg m/s^2 = 1 Newton = 0.2248 pounds force (pounds force is usually abbreviated lbf and Newton N) (equivalently 1 lbf = 4.448 N)
- Energy = force x length = mass x length^2/time^2
    - 1 kg m^2/s^2 = 1 Joule (J)
    - 778 ft lbf = 1 British thermal unit (BTU)
    - 1055 J = 1 BTU
    - 1 J = 0.7376 ft lbf
    - 1 calorie = 4.184 J
    - 1 dietary calorie = 1000 calories
- Power (energy/time = mass x length^2/time^3)
    - 1 J/s = 1 kg m^2/s^3 = 1 Watt
    - 746 W = 550 ft lbf/sec = 1 horsepower
- Heat capacity = J/moleK or J/kgK or J/mole°C or J/kg°C (see note below)
- Pressure = force/area
    - 1 N/m^2 = 1 Pascal
    - 101325 Pascal = 101325 N/m^2 = 14.696 lbf/in^2 = 1 standard atmosphere
- Current = charge/time (1 amp = 1 coul/s)
- Voltage = energy/charge (1 Volt = 1 J/coul)
- Capacitance = amps / (volts/s) (1 farad = 1 coul^2/J)
- Inductance = volts / (amps/s) (1 Henry = 1 J s^2 / coul^2)
- Resistance = volts/amps (1 ohm = 1 volt/amp = 1 Joule s / coul^2)
- Torque = force x lever arm length = mass x length^2/time^2 – same as energy but one would usually report torque in Nm (Newton meters), not Joules, to avoid confusion.
- Radians, degrees, revolutions – these are all dimensionless quantities, but must be converted between each other, i.e. 1 revolution = 2π radians = 360 degrees.

Special consideration 1: pounds force vs. pounds mass

توجه، ملاحظه lbf lbm

By far the biggest problem with USCS units is with mass and force. The problem is that pounds is both a unit of mass AND force. These are distinguished by lbm for pounds (mass) and lbf for pounds (force). We all know that W = mg where W = weight, m = mass, g = acceleration of gravity. So

گرانش تمایز شتاب

$$1\ \text{lbf} = 1\ \text{lbm x g} = 32.174\ \text{lbm ft/s}^2 \qquad \text{(Equation 3)}$$

Sounds ok, huh? But wait, now we have an extra factor of 32.174 floating around. Is it also true that

$$1\ \text{lbf} = 1\ \text{lbm ft/s}^2$$

which is analogous to the SI unit statement that

$$1\ \text{Newton} = 1\ \text{kg m/s}^2 \qquad \text{(Equation 4)}$$

No, 1 lbf cannot equal 1 lbm ft/s^2 because 1 lbf equals 32.174 lbm ft/sec^2. So what unit of mass satisfies the relation

$$1\ \text{lbf} = 1\ \text{(mass unit) ft/s}^2\text{?}$$

This mass unit is called a "slug" believe it or not. With use of equation (2) it is apparent that

$$1\ \text{slug} = 32.174\ \text{lbm} = 14.59\ \text{kg} \qquad \text{(Equation 5)}$$

Often when doing USCS conversions, it is convenient to introduce a conversion factor called g_c; by rearranging Equation 3 we can write

$$g_c = \frac{32.174\ \text{lbm ft}}{\text{lbf s}^2} = 1 \qquad \text{(Equation 6).}$$

Since Equation 2 shows that $g_c = 1$, one can multiply and divide any equation by g_c as many times as necessary to get the units into a more compact form *(an example of "why didn't somebody just say that?").* Keep in mind that **any** units conversion is simply a matter of multiplying or dividing by 1, e.g.

$$\frac{5280\ \text{ft}}{\text{mile}} = 1;\ \frac{1\ \text{kg m}}{\text{N s}^2} = 1;\ \frac{778\ \text{ft lbf}}{\text{BTU}} = 1;\ \textit{etc.}$$

For some reason 32.174 lbm ft/ lbf s^2 has been assigned a special symbol called g_c even though there are many other ways of writing 1 (e.g. 5280 ft / mile, 1 kg m / N s^2, 778 ft lbf / BTU) all of which are also equal to 1 but none of which are assigned special symbols.

If this seems confusing, I don't blame you. That's why I recommend that even for problems in which the givens are in USCS units and where the answer is needed in USCS units, first convert everything to SI units, do the problem, then convert back to USCS units. I disagree with some authors who say an engineer should have "native fluency" in both systems; it is somewhat useful but not necessary. The second example in the next sub-section below uses the approach of converting to SI, do the problem, and convert back to USCS. The third example shows the use of USCS units employing g_c.

Special consideration 2: temperature

Many difficulties also arise with units of temperature. There are four temperature مقیاس scales in "common" use: Fahrenheit, Rankine, Celsius (or Centigrade) and Kelvin. Note that one speaks of

درجه فارنهایت درجه سلسیوس

“degrees Fahrenheit” and “degrees Celsius” but just “Rankines” or “Kelvins” (without the “degrees”).

T (in units of °F) = T (in units of R) - 459.67
T (in units of °C) = T (in units of K) - 273.15
1 K = 1.8 R
T (in units of °C) = [T (in units of °F) – 32]/1.8,
T (in units of °F) = 1.8[T (in units of °C)] + 32
Water freezes at 32°F / 0°C, boils at 212°F / 100°C

Special note *(another example of “that’s so easy, why didn’t somebody just say that?”)*: when using units involving temperature (such as heat capacity, units J/kg°C, or thermal conductivity, units Watts/m°C), one can convert the temperature in these quantities these to/from USCS units (e.g. heat capacity in BTU/lbm°F or thermal conductivity in BTU/hr ft °F) simply by multiplying or dividing by 1.8. You don’t need to add or subtract 32. Why? Because these quantities are really derivatives with respect to temperature (heat capacity is the derivative of internal energy with respect to temperature) or refer to a temperature gradient (thermal conductivity is the rate of heat transfer per unit area by conduction divided by the temperature gradient, dT/dx). When one takes the derivative of the constant 32, you get zero. For example, if the temperature changes from 84°C to 17°C over a distance of 0.5 meter, the temperature gradient is (84-17)/0.5 = 134°C/m. In Fahrenheit, the gradient is [(1.8*84 +32) – (1.8*17 + 32)]/0.5 = 241.2°F/m or 241.2/3.281 = 73.5°F/ft. The important point is that the 32 cancels out when taking the difference. So *for the purpose of converting between °F and °C in units like heat capacity and thermal conductivity,* one can use 1°C = 1.8°F. That **doesn’t** mean that one can just skip the + or – 32 whenever one is lazy.

Also, one often sees thermal conductivity in units of W/m°C or W/mK. How does one convert between the two? Do you have to add or subtract 273? And how do you add or subtract 273 when the units of thermal conductivity are not degrees? Again, thermal conductivity is heat transfer per unit area **per unit temperature gradient**. This gradient could be expressed in the above example as (84°C-17°C)/0.5 m = 134°C/m, or in Kelvin units, [(84 + 273)K – (17 + 273)K]/0.5 m = 134K/m and thus the 273 cancels out. So one can say that 1 W/m°C = 1 W/mK, or 1 J/kg°C = 1 J/kgK. And again, that **doesn’t** mean that one can just skip the + or – 273 (or 460, in USCS units) whenever one is lazy.

Examples of the use (and power) of units

Example 1

An object has a weight of 300 lbf at earth gravity. What is its mass in units of lbm?

$$F = ma \Rightarrow m = \frac{F}{a} = \frac{F}{a}(1) = \frac{F}{a}(g_c) = \frac{300\ lbf}{32.174\frac{ft}{s^2}}\left(\frac{32.174\ lbm\ ft}{lbf\ s^2}\right) = 300\ lbm$$

This shows that an object that weighs 300 lbf at earth gravity has a mass of 300 lbm. At any other gravity level, its mass would still be 300 lbm but its weight would be different, but in all cases this weight would still be calculated according to F = ma (force = mass x acceleration) or, specifically for weights, we can use W = mg (weight = mass x acceleration of gravity).

Example 2

What is the weight (in lbf) of one gallon of air at 1 atm and 25˚C? The molecular mass of air is 28.97 g/mole = 0.02897 kg/mole.

Ideal gas law: $PV = n\Re T$

(P = pressure, V = volume, n = number of moles, $\Re$ = universal gas constant, T = temperature)

Mass of gas (m) = moles x mass/mole = $n\mathcal{M}$ ($\mathcal{M}$ = molecular mass)

Weight of gas (W) = mg, where g = acceleration of gravity = 9.81 $\mathrm{m/s^2}$

Combining these 3 relations: $W = PV\mathcal{M}g/\Re T$

$$W = \frac{PVMg}{\Re T} = \frac{\left(1\ \mathrm{atm}\frac{101325\ \mathrm{N/m^2}}{\mathrm{atm}}\right)\left(1\ \mathrm{gal}\frac{\mathrm{ft}^3}{7.481\ \mathrm{gal}}\left(\frac{\mathrm{m}}{3.281\ \mathrm{ft}}\right)^3\right)\left(\frac{0.02897\ \mathrm{kg}}{\mathrm{mole}}\right)\left(\frac{9.81\ \mathrm{m}}{\mathrm{s}^2}\right)}{\left(\frac{8.314\ \mathrm{J}}{\mathrm{mole\ K}}\right)(25+273)\mathrm{K}}$$

$$W = 0.0440\frac{\left(\frac{\mathrm{N}}{\mathrm{m}^2}\right)\left(\mathrm{m}^3\right)\left(\frac{\mathrm{kg}}{\mathrm{mole}}\right)\left(\frac{\mathrm{m}}{\mathrm{s}^2}\right)}{\left(\frac{\mathrm{J}}{\mathrm{mole\ K}}\right)\mathrm{K}} = 0.0440\frac{(\mathrm{N})(\mathrm{m})(\mathrm{kg})\left(\frac{\mathrm{m}}{\mathrm{s}^2}\right)}{\mathrm{J}} = 0.0440\frac{(\mathrm{N})\left(\frac{\mathrm{kg\ m}^2}{\mathrm{s}^2}\right)}{\mathrm{J}}$$

$$W = 0.0440\ \mathrm{N}\left(\frac{1\ \mathrm{lbf}}{4.448\ \mathrm{N}}\right) = 0.00989\ \mathrm{lbf} \approx\ 0.01\ \mathrm{lbf}$$

Note that it's easy to write down all the formulas and conversions. The tricky part is to check to see if you've actually gotten all the units right. In this case I converted everything to the SI system first, then converted back to USCS units at the very end – which is a pretty good strategy for most problems. The tricky parts are realizing (1) the temperature must be an absolute temperature, i.e. Kelvin not ˚C, and (2) that moles are not the same as mass, so you have to convert using $\mathcal{M}$. If in doubt, how do you know whether to multiply or divide by $\mathcal{M}$? Check the units!

Example 3

A car with a mass of 3000 lbm is moving at a velocity of 88 ft/s. What is its kinetic energy (KE) in units of ft lbf? What is its kinetic energy in Joules?

$$\text{KE} = \frac{1}{2}(\text{mass})(\text{velocity})^2 = \frac{1}{2}(3000\ \text{lbm})\left(88\frac{\text{ft}}{\text{s}}\right)^2 = 1.16 \text{ x } 10^7\ \frac{\text{lbm ft}^2}{\text{s}^2}$$

Now what can we do with lbm ft^2/s^2??? The units are (mass)$(\text{length})^2/(\text{time})^2$, so it is a unit of energy, so at least that part is correct. Dividing by g_c, we obtain

$$\text{KE} = \left(1.16 \text{ x } 10^7\ \frac{\text{lbm ft}^2}{\text{s}^2}\right)\left(\frac{1}{g_c}\right) = \left(1.16 \text{ x } 10^7\ \frac{\text{lbm ft}^2}{\text{s}^2}\right)\left(\frac{\text{lbf s}^2}{32.174\ \text{lbm ft}}\right) = 3.61 \text{ x } 10^5\ \text{ft lbf}$$

$$\text{KE} = \left(3.61\times10^5\ \text{ft lbf}\right)\left(\frac{1\ \text{J}}{0.7376\ \text{ft lbf}}\right) = 4.89\times10^5\ \text{J}$$

Note that if you used 3000 lbf rather than 3000 lbm in the expression for KE, you'd have the wrong units – ft lbf^2/lbm, which is NOT a unit of energy (or anything else that I know of…) Also note that since $g_c = 1$, we COULD multiply by g_c rather than divide by g_c; the resulting units ($\text{lbm}^2\ \text{ft}^3/\text{lbf s}^4$) is still a unit of energy, but not a very useful one!

Example 4

The thermal conductivity of a particular brand of ceramic insulating material is $0.5\frac{\text{BTU in}}{\text{ft}^2\ \text{hr °F}}$ (I'm not kidding, these are the units commonly reported in commercial products!) where the standard abbreviations in = inch and hr = hour are used. What is the thermal conductivity in units of $\frac{\text{W}}{\text{m °C}}$? (Here "W" = Watt, not weight.)

$$\left(0.5\frac{\text{BTU in}}{\text{ft}^2\ \text{hr °F}}\right)\left(\frac{1055\ \text{J}}{\text{BTU}}\right)\left(\frac{\text{ft}}{12\text{in}}\right)\left(\frac{3.281\ \text{ft}}{\text{m}}\right)\left(\frac{\text{hr}}{3600\ \text{s}}\right)\left(\frac{1\ \text{W}}{1\ \text{J/s}}\right)\left(\frac{1.8\ \text{°F}}{\text{°C}}\right) = 0.0721\frac{\text{W}}{\text{m °C}}$$

Note that the thermal conductivity of air at room temperature is 0.026 Watt/m˚C, *i.e.*, about 3 times lower than the insulation. So why don't we use air as an insulator? We'll discuss that in Chapter 7.

مهندسی بررسی موشکافانه

Chapter 3. "Engineering scrutiny"

"Be your own worst critic, unless you prefer that someone else be your worst critic."
- I dunno, I just made it up. But, it doesn't sound very original.

don't know

حاسبوا قبل ان تحاسبوا
به حساب خود برسید قبل از آنکه به حسابتان رسیدگی شود.

Scrutinizing analytical formulas and results

I often see analyses that I can tell within 5 seconds must be wrong. I have three tests, which should be done in the order listed, for checking and verifying results. These tests will weed out 95% of all mistakes. I call these the "smoke test," "function test," and "performance test," by analogy with building electronic devices.

1. *Smoke test.* In electronics, this corresponds to turning the power switch on and seeing if the device smokes or not. If it smokes, you know the device can't possibly be working right (unless you intended for it to smoke.) In analytical engineering terms, this corresponds to **checking the units**. You have no idea how many results people report that can't be correct because the units are wrong (i.e. the result was 6 kilograms, but they were trying to calculate the speed of something.) **You will catch 90% of your mistakes if you just check the units.** For example, if I just derived the ideal gas law for the first time and predicted PV = n$\Re$/T you can quickly see that the units on the right-hand side of the equation are different from those on the left-hand side. There are several additional rules that must be followed:

   چک کردن آحاد

   - Anything inside a square root, cube root, etc. must have units that are a perfect square (e.g. m^2/sec^2), cube, etc.) This does **not** mean that every term inside the square root must be a perfect square, only that the **combination** of all terms must be a perfect square. For example, the speed (v) of a frictionless freely falling object in a gravitational field is $v = \sqrt{2gh}$, where g = acceleration of gravity (units length/time^2) and h is the height from which the object was dropped (units length). Neither g nor h have units that are a perfect square, but when multiplied together the units are (length/time^2)(length) = length^2/time^2, which is a perfect square, and when you take the square root, the units are $v = \sqrt{length^2/time^2} = length/time$ as required.
   - Anything inside a log, exponent, trigonometric function, etc., must be dimensionless (I can take the log of 6 but I don't know how to take the log of 6 kilograms). Again, the individual terms inside the function need not all be dimensionless, but the combination must be dimensionless.
   - Any two quantities that are added together must have the same units (I can't add 6 kilograms and 19 meters/second. Also, I can add 6 miles per hour and 19 meters per second, but I have to convert 6 miles per hour into meters per second, or convert 19 meters per second into miles per hour, before adding the terms together.)

2. *Function test.* In electronics, this corresponds to checking to see if the device does what I designed it to do, e.g. that the red light blinks when I flip switch on, the meter reading increases when I turn the knob to the right, the bell rings when I push the button, etc. – assuming that was what I intended that it do. In analytical terms this corresponds to determining if the result gives sensible predictions. Again, there are several rules that must be followed:

   طراحی

   پیش بینی محسوس

a ← nominator صورت کسر
b ← denominator مخرج کسر

- Determine تعیین کردن if the sign علامت (+ or -) of the result is reasonable منطقی. For example, if your prediction پیش‌بینی of the absolute مطلق temperature دما of something is –72 Kelvin, you should check your analysis again.
- For terms in an equation معادله، رابطه with property values in the denominator, can that value be zero? (In which case the term would go to infinity بی‌نهایت). Even if the property can't go to zero, does it make sense that as the value decreases, the term would increase?
- Determine whether what happens to y as x goes up or down is reasonable or not. For example, in the ideal gas law, PV = n$\Re$T:
    - At fixed volume (V) and number of moles of gas (n), as T increases then P increases – reasonable
    - At fixed temperature (T) and n, as V increases then P decreases – reasonable
    - Etc.
- Determine what happens in the limit where x goes to special values, e.g. zero, one or infinity as appropriate. For example, consider the equation for the temperature as a function of time T(t) of an شیء object starting at temperature T_i at time t = 0 having surface area A (units m^2), volume V (units m^3), density ρ (units kg/m^3) and heat capacity C_P (units J/kg°C) that is suddenly dunked into a fluid at temperature T_∞ with heat transfer coefficient h (units Watts/m^2°C). It can be shown that in this case T(t) is given by

$$T(t)=T_\infty+(T_i-T_\infty)\exp\left(-\frac{hA}{\rho V C_P}\overset{-\infty}{\uparrow} t\right) \qquad \text{(Equation 7)}$$

$t \to \infty \Rightarrow T(\infty) = T_\infty$

$\to e^{-\infty} \to \frac{1}{e^{\infty}} \approx 0$

hA/ρC_P has units of (Watts/m^2°C)(m^2)/(kg/m^3)(m^3)(J/kg°C) = 1/s, so (hA/ρVC_P)t is dimensionless, thus the formula easily passes the smoke test. But does it make sense? At t = 0, T_i = 0 as expected. What happens if you wait for a long time? The temperature can reach T_∞ but cannot overshoot it (a consequence of the Second Law of Thermodynamics, discussed in Chapter 7). In the limit $t \to \infty$, the term exp(-(hA/ρVC_P)t) goes to zero, thus $T \to T_\infty$ as expected. Other scrutiny checks: if h or A increases, heat can be transferred to the object more quickly, thus the time to approach T_∞ decreases. Also, if ρ, V or C_P increases, the "thermal inertia" اینرسی حرارتی (resistance to change in temperature) increases, so the time required to approach T_∞ increases. So, the formula makes sense.

- If your formula contains شامل a difference of terms, determine what happens if those 2 terms are equal. For example, in the above formula, if $T_i = T_\infty$, then the formula becomes simply T(t) = T_∞ for all time. This makes sense because if the bar temperature and fluid temperature are the same, then there is no heat transfer to or from the bar and thus its temperature never changes (again, a consequence of the Second Law of Thermodynamics … two objects at the same temperature cannot exchange energy via heat transfer.)

3. *Performance* عملکرد *test.* In electronics, this corresponds to determining how fast, how accurate دقیق, etc. the device is. In analytical terms this corresponds to determining how accurate the result is. This means of course you have to compare مقایسه it to something else **that you trust** اطمینان, i.e. an experiment, a more sophisticated پیچیده، پیشرفته analysis, someone else's published result (of course there is no guarantee ضمانت that their result is correct just because it got published, but you need to check it anyway.) For example, if I derived the ideal gas law and predicted PV = 7nRT, it passes the smoke and function tests with no problem, but it fails the performance test miserably به طرز ناراحت کننده (by a factor of 7). But

of course the problem is deciding which result to trust as being at least as accurate as your own result; this of course is something that cannot be determined in a rigorous way, it requires a judgment call based on your experience.

تصمیم سختگیرانه قضاوت تجربه

Scrutinizing computer solutions

(This part is beyond what I expect you to know for AME 101 but I include it for completeness).

مهندسی

[illegible] عددی

Similar to analyses, I often see computational results that I can tell within 5 seconds must be wrong. **It is notoriously easy to be lulled into a sense of confidence in computed results, because the computer always gives you some result, and that result always looks good when plotted in a 3D shaded color orthographic projection.** The corresponding "smoke test," "function test," and "performance test," are as follows:

بدنام آرام به نظر رسیدن

Converge همگرا

Conserve // بقا [illegible]

1. *Smoke test.* Start the computer program running, and see if it crashes or not. If it doesn't crash, you've passed the smoke test, part (a). Part (b) of the smoke test is to determine if the computed result passes the *global conservation test.* The goal of any program is to satisfy mass, momentum, energy and atom conservation *at every point in the computational domain* subject to certain constituitive relations (*e.g.*, Newton's law of viscosity $\tau_x = \mu\partial u_x/\partial y$), Hooke's Law $\sigma = E\varepsilon$) and equations of state (*e.g.*, the ideal gas law.) This is a hard problem, and it is even hard to verify that the solution is correct once it is obtained. But it is easy to determine whether or not *global conservation* is satisfied, that is,

در هم شکستن

راضی در برقرار پایستگی کلی

شار جرم

- Is mass conserved, that is, does the sum of all the mass fluxes at the inlets, minus the mass fluxes at the outlets, equal to the rate of change of mass of the system (=0 for steady problems)?
- Is momentum conserved in each coordinate direction?
- Is energy conserved?
- Is each type of atom conserved?

جهت مختصات

محاسبات

If not, you are 100% certain that your calculation is wrong. You would be amazed at how many results are never "sanity checked" in this way, and in fact fail the sanity check when, after months or years of effort and somehow the results never look right, someone finally gets around to checking these things, the calculations fail the test and you realize all that time and effort was wasted.

سلامت عقل تلاش فهمیدن هدر رفتن

validation

صحت سنجی

2. *Performance test.* Comes before the function test in this case. For computational studies, a critical performance test is to *compare your result to a known analytical result under simplified conditions.* For example, if you're computing flow in a pipe at high Reynolds numbers (where the flow is turbulent), with chemical reaction, temperature-dependent transport properties, variable density, etc., first **check your result against the textbook solution** that assumes constant density, constant transport properties, etc., by making all of the simplifying assumptions (in your model) that the analytical solution employs. **If you don't do this, you really have no way of knowing if your model is valid or not.** You can also use previous computations by yourself or others for testing, but of course there is no absolute guarantee that those computations were correct.

3. *Function test.* Similar to function test for analyses.

By the way, even if you're just doing a quick calculation, I recommend **not** using a calculator. Enter the data into an Excel spreadsheet so that you can add/change/scrutinize/save calculations as needed. Sometimes I see an obviously invalid result and when I ask, "How did you get that result? What numbers did you use?" the answer is "I put the numbers into the calculator and this was the result I got." But how do you know you entered the numbers and formulas correctly? What if you need to re-do the calculation for a slightly different set of numbers?

اندکی، کمی

Examples of the use of units and scrutiny

These examples, particularly the first one, also introduce the concept of "back of the envelope" (that is, simple, approximate but instructive) estimates, a powerful engineering tool.

آموزنده ، تخمین ، ابزار

Example 1. Drag force and power requirements for an automobile

ضریب پسا ، تعریف

A car with good aerodynamics has a drag coefficient (C_D) of 0.3. The drag coefficient is defined as the ratio of the drag force (F_D) to the *dynamic pressure* of the flow = $\frac{1}{2}\rho v^2$ (where ρ is the fluid density and v the fluid velocity far from the object) multiplied by the cross-section area (A) of the object, *i.e.*

$$F_D = \frac{1}{2} C_D \rho v^2 A \qquad \text{(Equation 8)}$$

$\frac{1}{2}$ → one half → one over two — یک دوم

i.e. → That is = id est — یعنی

The density of air at standard conditions is 1.18 kg/m³.

(a) Estimate the power (in units of horsepower) required to overcome the aerodynamic drag of such a car at 60 miles per hour.

$$P = Fv \ \left(P = \text{power},\ F = \text{force},\ v = \text{velocity}\right);\ v = \frac{60\ mi}{hr}\frac{5280\,ft}{mi}\frac{m}{3.281\,ft}\frac{hr}{60\,\text{min}}\frac{\text{min}}{60\ s} = 26.8\frac{m}{s}$$

Estimate the cross-section area of the car as 2 m x 1.5 m = 3 m²

$$F_D = \frac{1}{2}C_D\rho v^2 A = \frac{1}{2}(0.3)\left(\frac{1.18\ kg}{m^3}\right)\left(26.8\frac{m}{s}\right)^2\left(3\ m^2\right) = 381.4\frac{kg\ m}{s^2} = 381.4N$$

$$P = Fv = (381.4N)\left(\frac{26.8\ m}{s}\right) = 1.022\times10^4\frac{Nm}{s} = 1.022\times10^4\frac{J}{s} = \left(1.022\times10^4 W\right)\left(\frac{1\ hp}{746\ W}\right) = 13.7\ hp$$

which is reasonable.

بنزین

Petrol → British

Gasoline → American

مسافت پیموده

(b) Estimate the gas mileage of such a car. The heating value of gasoline is 4.3 x 10⁷ J/kg and its density is 750 kg/m³.

$$\frac{mi}{gal} = \frac{mi}{hr}\frac{hr}{gal} = \frac{mi}{hr}\frac{hr}{J}\frac{J}{gal} = \frac{mi}{hr}\left(\frac{s}{J}\frac{hr}{s}\right)\left(\frac{J}{kg}\frac{kg}{gal}\right) == \frac{mi}{hr}\frac{s}{J}\frac{hr}{s}\frac{J}{kg}\left(\frac{kg}{m^3}\frac{m^3}{ft^3}\frac{ft^3}{gal}\right)$$

واحد حجم

Gallon < US : 3.785 L
British : 4.546 L

1 oil barrel = 42 US Gallon ≃ 159 L

یک بشکه نفت

معادل وزن یک بشکه نفت خام ≃ 136 kg

$$\frac{mi}{gal}=\left(\frac{60\ mi}{hr}\right)\left(\frac{s}{1.022\times10^{4}J}\right)\left(\frac{hr}{3600\ s}\right)\left(\frac{4.3\times10^{7}J}{kg}\right)\left(\frac{750\ kg}{m^{3}}\right)\left(\frac{m}{3.218\,ft}\right)^{3}\left(\frac{ft^{3}}{7.481\ gal}\right)$$

$$=199.0706726\frac{mi}{gal}$$

Why is this value of miles/gallon so high?

- The main problem is that conversion of fuel energy to engine output shaft work is about 20% efficient at highway cruise conditions, thus the gas mileage would be 199.0706726 x 0.2 = 39.81413452 mpg → مایل بر گالن
- Also, besides air drag, there are other losses in the transmission, driveline, tires – at best the drivetrain is 80% efficient – so now we're down to 31.85130761 mpg
- Also – other loads on engine – air conditioning, generator, …

قابل قبول

What else is wrong? There are too many significant figures; at most 2 or 3 are acceptable. When we state 31.85130761 mpg, that means we think that the miles per gallon is closer to 31.85130761 mpg than 31.85130760 mpg or 31.85130762 mpg. Of course we can't measure the miles per gallon to anywhere near this level of accuracy. 31 is probably ok, 31.9 is questionable and 31.85 is ridiculous. You will want to carry a few extra digits of precision through the calculations to avoid round-off errors, but then at the end, round off your calculation to a reasonable number of significant figures *based on the uncertainty of the most uncertain parameter*. That is, if I know the drag coefficient only to the first digit, i.e. I know that it's closer to 0.3 than 0.2 or 0.4, but not more precisely than that, there is no point in reporting the result to 3 significant figures.

ارقام، اشکال

Example 2. Scrutiny of a new formula

$\frac{\partial}{\partial t}$ نرخ

I calculated for the first time ever the rate of heat transfer (q) (in Watts) as a function of time t from an aluminum bar of radius r, length L, thermal conductivity k (units Watts/m˚C), thermal diffusivity α (units m^2/s), heat transfer coefficient h (units Watts/m^2°C) and initial temperature T_{bar} conducting and radiating to surroundings at temperature T_∞ as

$$q=k\left(T_{bar}-T_{\infty}\right)e^{\alpha t/r^{2}}-hr^{2}\left(T_{bar}-T_{\infty}-1\right) \qquad \text{(Equation 9)}$$

Using "engineering scrutiny," what "obvious" mistakes can you find with this formula? What is the likely "correct" formula?

1. The units are wrong in the first term (Watts/m, not Watts)

2. The units are wrong in the second term inside the parenthesis (can't add 1 and something with units of temperature)

3. The first term on the right side of the equation goes to infinity as the time (t) goes to infinity – probably there should be a negative sign in the exponent so that the whole term goes to zero as time goes to infinity.

4. The length of the bar (L) doesn't appear anywhere

Uncertainty عدم قطعیت

Precisely دقیق، به دقت

5. The signs on ($T_{bar} - T_\infty$) are different in the two terms – but heat must ALWAYS be transferred from hot to cold, never the reverse, so the two terms cannot have different signs. One can, with equal validity, define heat transfer as being positive either to or from the bar, but with either definition, you can't have heat transfer being positive in one term and negative in the other term.

6. Only the first term on the right side of the equation is multiplied by the $e^{(-\alpha t/r^2)}$ factor, and thus will go to zero as t → ∞. So the other term would still be non-zero even when t → ∞, which doesn't make sense since the amount of heat transfer (q) has to go to zero as t → ∞. So probably both terms should be multiplied by the $e^{(-\alpha t/r^2)}$ factor.

Based on these considerations, a possibly correct formula, which would pass all of the smoke and function tests is

$$q = \left[kL\left(T_{bar} - T_\infty\right) + hr^2\left(T_{bar} - T_\infty\right)\right]e^{-\alpha t/r^2}$$

متناسب

Actually even this is a bit odd since the first term (conduction heat transfer) is proportional to the length L but the second term (convection heat transfer) is independent of L … a still more likely formula would have both terms proportional to L, e.g.

مستقل

$$q = \left[kL\left(T_{bar} - T_\infty\right) + hrL\left(T_{bar} - T_\infty\right)\right]e^{-\alpha t/r^2}$$

Example 3. Thermoelectric generator

The thermal efficiency (η) = (electrical power out) / (thermal power in) of a thermoelectric power generation device (used in outer planetary spacecraft (Figure 2), powered by heat generated from radioisotope decay, typically plutonium-238) is given by

$$\eta = \left(1 - \frac{T_L}{T_H}\right)\frac{\sqrt{1+ZT_a} - 1}{\sqrt{1+ZT_a} + T_L/T_H}; T_a \equiv \frac{T_H + T_L}{2} \qquad \text{(Equation 10)}$$

where T is the temperature, the subscripts L, H and a refer to cold-side (low temperature), hot-side (high temperature) and average respectively, and Z is the "thermoelectric figure of merit":

$$Z \equiv \frac{S^2}{\rho k} \qquad \text{(Equation 11)}$$

where S is the *Seebeck coefficient* of material (units Volts/K, indicates how many volts are produced for each degree of temperature change across the material), ρ is the electrical resistivity (units ohm m) *(not to be confused with density!)* and k is the material's thermal conductivity (W/mK).

(a) show that the units are valid (passes smoke test)

Everything is obviously dimensionless except for ZT_a, which must itself be dimensionless so that I can add it to 1. Note

$$ZT_a = \frac{S^2}{\rho k} T_a = \frac{\left(\dfrac{Volt}{K}\right)^2}{(ohm\ m)\left(\dfrac{W}{m\ K}\right)} K = \frac{\left(\dfrac{J/coul}{K}\right)^2}{\left(\dfrac{J\ s}{coul^2}\ m\right)\left(\dfrac{J/s}{m\ K}\right)} K = \frac{J^2}{J^2} \frac{\left(\dfrac{1}{coul}\right)^2}{\left(\dfrac{1}{coul^2}\right)} \frac{1}{s\dfrac{1}{s}} \frac{\dfrac{1}{K^2}K}{\dfrac{1}{K}} = 1 \quad \text{OK}$$

(b) show that the equation makes physical sense (passes function test)

- If the material Z = 0, it produces no electrical power thus the efficiency should be zero. If Z = 0 then

$$\eta = \left(1 - \frac{T_L}{T_H}\right) \frac{\sqrt{1+0T_a}-1}{\sqrt{1+0T_a}+T_L/T_H} = \left(1 - \frac{T_L}{T_H}\right) \frac{\sqrt{1}-1}{\sqrt{1}+T_L/T_H} = \left(1 - \frac{T_L}{T_H}\right) \frac{0}{1+T_L/T_H} = 0 \qquad \text{OK}$$

- If $T_L = T_H$, then there is no temperature difference across the thermoelectric material, and thus no power can be generated. In this case

$$\eta = (1-1)\frac{\sqrt{1+ZT_a}-1}{\sqrt{1+ZT_a}+T_L/T_H} = (0)\frac{\sqrt{1+ZT_a}-1}{\sqrt{1+ZT_a}+T_L/T_H} = 0 \qquad \text{OK}$$

- Even the best possible material ($ZT_a \to \infty$) cannot produce an efficiency greater than the theoretically best possible efficiency (called the *Carnot cycle* efficiency, see page 91) = $1 - T_L/T_H$, for the same temperature range. As $ZT_a \to \infty$,

$$\eta \approx \left(1 - \frac{T_L}{T_H}\right) \frac{\sqrt{ZT_a}-1}{\sqrt{ZT_a}+T_L/T_H} \approx \left(1 - \frac{T_L}{T_H}\right) \frac{\sqrt{ZT_a}}{\sqrt{ZT_a}} = 1 - \frac{T_L}{T_H} \qquad \text{OK}$$

Side note #1: a good thermoelectric material such as Bi_2Te_3 has $ZT_a \approx 1$ and works up to about 200°C before it starts to melt, thus

$$\eta = \left(1 - \frac{T_L}{T_H}\right) \frac{\sqrt{1+1}-1}{\sqrt{1+1}+(25+273)/(200+273)} = 0.203\left(1 - \frac{T_L}{T_H}\right) = 0.203\ \eta_{Carnot}$$

$$= 0.203\left(1 - \frac{25+273}{200+273}\right) = 0.0750 = 7.50\%$$

By comparison, your car engine has an efficiency of about 25%. So practical thermoelectric materials are, in general, not very good sources of electrical power, but are extremely useful in some niche applications, particularly when either (1) it is essential to have a device with no moving parts or (2) a "free" source of thermal energy at relatively low temperature is available, *e.g.*, the exhaust of an internal combustion engine.

Side note #2: a good thermoelectric material has a high S, so produces a large voltage for a small temperature change, a low ρ so that the resistance of the material to the flow of electric current is low, and a low k so that the temperature across the material ΔT is high. The heat transfer rate (in Watts) $q = kA\Delta T/\Delta x$ (see Chapter 7) where A is the cross-sectional area of the material and Δx is its thickness. So for a given ΔT, a smaller k means less q is transferred across the material. One might think that less q is worse, but no. Consider this:

The electrical power $= IV = (V/R)V = V^2/R = (S\Delta T)^2/(\rho\Delta x/A) = S^2\Delta T^2 A/\rho\Delta x$.

The thermal power $= kA\Delta T/\Delta x$

The ratio of electrical to thermal power is $[S^2\Delta T^2 A/\rho\Delta x]/[kA\Delta T/\Delta x] = (S^2/\rho k)\Delta T = Z\Delta T$, which is why Z is the "figure of merit" for thermoelectric generators.)

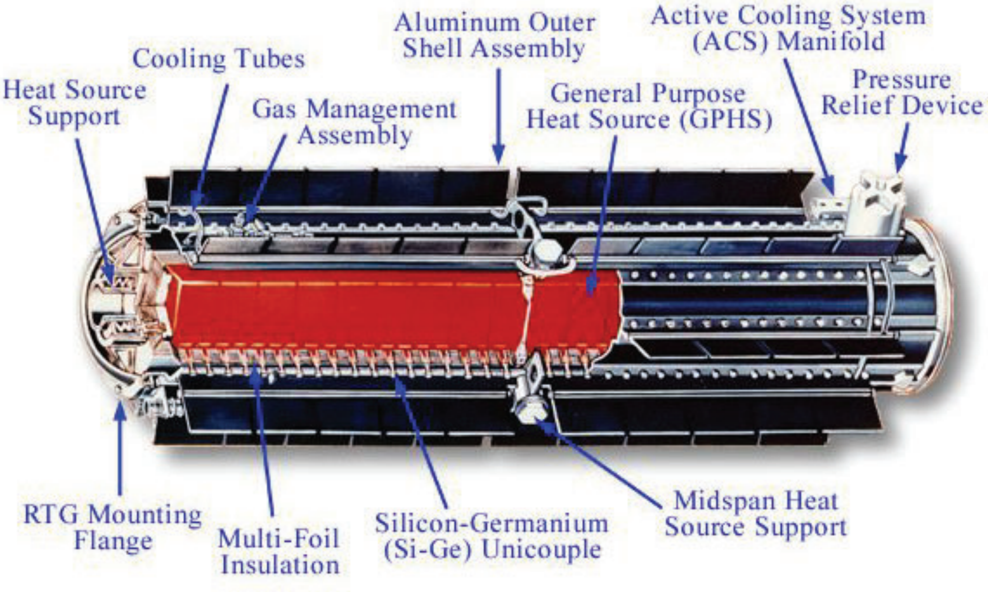

Figure 2. Radioisotope thermoelectric generator used for deep space missions. Note that the plutonium-238 radioisotope is called simply, "General Purpose Heat Source."

Example 4. Density of matter

Estimate the density of a neutron. Does the result make sense? The density of a white dwarf star is about 2×10^{12} kg/m^3 – is this reasonable?

The mass of a neutron is about one atomic mass unit (AMU), where a carbon-12 atom has a mass of 12 AMU and a mole of carbon-12 atoms has a mass of 12 grams. Thus one neutron has a mass of

$$(1\text{ AMU})\left(\frac{1\text{ C-12 atom}}{12\text{ AMU}}\right)\left(\frac{1\text{ mole C-12}}{6.02\times10^{23}\text{ atoms C-12}}\right)\left(\frac{12\text{ g C-12}}{\text{mole C-12}}\right)\left(\frac{1\text{ kg}}{1000\text{ g}}\right)=1.66\times10^{-27}\text{kg}$$

A neutron has a radius (r) of about 0.8 femtometer = 0.8×10^{-15} meter. Treating the neutron as a sphere, the volume is $4\pi r^3/3$, and the density (ρ) is the mass divided by the volume, thus

$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{1.66\times10^{-27}\text{kg}}{\frac{4\pi}{3}\left(0.8\times10^{-15}\text{m}\right)^3} = 7.75\times10^{17}\,\frac{\text{kg}}{\text{m}^3}$$

By comparison, water has a density of 10^3 kg/m^3, so the density of a neutron is far higher (by a factor of 10^{14}) than that of atoms including their electrons. This is expected since the nucleus of an atom occupies only a small portion of the total space occupied by an atom – most of the atom is empty space where the electrons reside. Also, even the density of the white dwarf star is far less than the neutrons (by a factor of 10^5), which shows that the electron structure is squashed by the mass of the star, but not nearly down to the neutron scale (protons have a mass and size similar to neutrons, so the same point applies to protons too.)